numbers, though seemingly complex, are, when mastered, extremely simple, and will often be found to be much quicker than the ordinary multiplication, with less liability to error.

In the first of these methods the number to be squared is considered to be the sum of an odd multiple of 25 and a number equal to or less than 50.

Rule: To the product of the two multiples of 50 next higher and lower than the multiple of 25, add 100 times the product of the number into the quotient of the higher multiple of 50 divided by 50. Then add the square of the difference between 25 and the number.

Examples.

\[
\begin{align*}
298 & = 275 + 23 \\
300 \times 250 & = 75000 \\
300 \times 100 \times 23 & = 138000 \\
2 \times 2 & = 4 \\
(298)^2 & = 88804 \\
412 & = 375 + 37 \\
400 \times 350 & = 140000 \\
400 \times 100 \times 37 & = 296000 \\
(37 - 25)^2 & = 144 \\
(412)^2 & = 169744
\end{align*}
\]

When the number lies between 25 and 75 the process reduces to this:

To the square of the difference between the number and 50 add as many hundreds as the number differs from 25.

Required the square of 67.

\[
\begin{align*}
(67 - 50)^2 & = 289 \\
(67 - 25) \times 100 & = 4200 \\
(67)^2 & = 4489
\end{align*}
\]

An algebraic statement of this rule is thus:

\[
(25y + x)^2 = 25(y + 1) \cdot 25(y - 1) + \frac{25(y + 1) \cdot 100x + (25 - x)^2}{50}
\]

where \(y\) is an odd number and \(x\) is less than 50.

An incidental rule in connection with this method is this:

The square of any odd multiple of 25 is equal to the product of the next higher and next lower multiples of 50, plus 625.

A second method is to consider the number to be made up of a number of hundreds plus a number less than 100.

Rule: Square the hundreds figure, subtract 1 from the result, and multiply by 10,000. Then add 1 to the hundreds figure and multiply by 200; multiply by this product the tens and units part of the given number. Next square what the given number lacks of the next hundred. Add these results together, and the sum will be the required square.

Examples:

\[
\begin{align*}
487 & = 400 + 87 \\
(4^2 - 1) \times 10000 & = 150000 \\
2(4 + 1) \times 100 \times 87 & = 87000 \\
(100 - 87)^2 & = 169 \\
(487)^2 & = 237169 \\
293 & = 200 + 93 \\
(2^2 - 1) \times 10000 & = 30000 \\
2(2 + 1) + 100 \times 93 & = 55800 \\
(100 - 93)^2 & = 49 \\
(293)^2 & = 85849
\end{align*}
\]

When the number lies between 100 and 200 the first product disappears, and the rule becomes:

To the square of the difference between the number and 200 add 400 times the difference between the number and 100.

\[
\begin{align*}
192 & = 100 + 92 \\
(200 - 192)^2 & = 64 \\
400 \times 92 & = 36800 \\
(192)^2 & = 36864
\end{align*}
\]

An algebraic statement of this rule is thus:

\[
(100y + x)^2 = 10000(y^2 - 1) + 200(y + 1)x + (100 - x)^2
\]

The first of these rules is especially useful in squaring numbers between 25 and 75, but is very easy of application to larger numbers.

The second rule is most useful in squaring numbers lying in the last quarter of the hundred.

To be able to use these rules readily, the squares of the numbers from 1 to 25 should be committed to memory.

E. G. T., '87.